Corrected Formula for the Calculation of the Electrical Heart Axis

Dragutin Novosel, Georg Noll1, Thomas F. Lüscher1
Outpatient Psychiatry Hospital Aargau, Baden; and 1Department of Cardiology, Zürich University Hospital, Zürich, Switzerland

The calculation of the heart axis in the frontal plane can be performed with the combination of any two leads. The use of combination of bipolar (I, II, III) and unipolar leads (aVR, aVL and aVF) can produce wrong results. Calculation of the electrical axis from leads I and aVF without correction (sometimes used in ECG recorders): $EA=\pm \text{Arctan} \left( \frac{\text{aVF}}{\text{I}} \right)$ results in lower values (in our study: $34^\circ \pm 4^\circ$, $n = 48$) as compared to the values obtained with formula that uses leads I and II: $EA=\pm \text{Arctan} \left( \frac{(2\times \text{II}-\text{I})}{(\text{Sqr}(3)\times \text{I})} \right)$ (axis = $37^\circ \pm 3^\circ$, $n=48$; $p<0.005$, paired t-test with Bonferroni correction) or with corrected formula which uses leads I and aVF: $EA=\pm \text{Arctan} \left( \frac{(2\times \text{aVF})}{(\text{Sqr}(3)\times \text{I})} \right)$ (axis = $37^\circ \pm 4^\circ$, $n=48$; $p<0.005$, paired t-test with Bonferroni correction). The correction factor $2/\text{Sqr}(3)$ is required because the unipolar and bipolar leads have different strengths. Although the difference rarely reach clinical significance, our results suggests that the ECG recorders should be proofed on formulas used for the calculation of the electrical axis.

Key words: computer data processing; computer use training; diagnosis, computer-assisted; ECG; electrocardiography; heart; software

The determination of the heart’s electrical axis (EA) is an important part of the electrocardiographic (ECG) diagnosis. The alteration of the EA can be seen in many pathological conditions, such as branch blocks and hypertrophies. Teaching basics of the ECG to the medical students during preclinical education always includes the determination of the EA (1).

Our hypothesis was that the EA calculated from the leads I and aVF would result in different values when compared with the results from the calculation using leads I and II. In the second step we tested our formula and compared it with the previously published formulas (2,3). Since only a few reports on this topic exist (2-5), some of which might results in erroneous interpretations (3), we tried to recalculate the EA with a validated method.

Methods
ECGs ($N=48$) were recorded with a 6 channel electrocardiograph (Schiller AT6, Schiller Reomed AG, Baar, Switzerland) enabling automatic printout of the QRS amplitudes. The patients were selected from the ECG laboratory of the University Hospital in Zürich. The ECGs were analyzed by an experienced cardiologist who was blinded about the aim of the study. For the purpose of this study, EA was calculated from the sum of the Q, R and S peaks (6,7). Normal EA was defined as the value between 0 and 90 degrees.

Statistical analysis was performed using paired t-tests. As we performed several tests on the same set of the data, the Bonferroni correction for single $p$ values was applied (8).

Bipolar limb leads measure the potential difference between two points. The most commonly used bipolar leads are those introduced by Einthoven (9). Three bipolar limb leads, denoted I, II and III, can be represented as follows:

$I = EL – ER$ [1]
$II = EF – ER$ [2]
$III = EF – EL$ [3]

where EL, ER, and EF denote the potential at the left and right arms and left leg, respectively (e.g., lead I measures the potential difference between the left and right arm). Formula [1] minus formula [2] gives the following equation:

$I-\text{II}=EL – ER – (EF – ER)$ [4]
$I-\text{II}=EL – EF$ [5]

The sum of the formula [5] and formula [3] results in the following, well known equation:

$I-\text{II}+\text{III}=EL – EF+EF – EL$ [6]
$I-\text{II}+\text{III}=0$ [7]

Augmented unipolar leads, introduced by Goldberg (10), are defined as the difference in potentials between one limb and the average of the two other limbs.
These augmented unipolar leads can be represented with the following equations:

\[ a_{VR} = ER - \frac{1}{2} (EL + EF) \]  
\[ a_{VL} = EL - \frac{1}{2} (EF + ER) \]  
\[ a_{VF} = EF - \frac{1}{2} (EL + ER) \]

The sum of the equations [8], [9], and [10] results in the following [12], also well known equation (7):

\[ a_{VR} + a_{VF} + a_{VF} = ER - \frac{1}{2}(EL + EF) + EL - \frac{1}{2}(EF + ER) + EF - \frac{1}{2}(EL + ER) \]

\[ a_{VR} + a_{VF} + a_{VF} = 0 \]  

The limb leads are closely related: if any two of them are known, the other four can be derived from them. To calculated aVF lead from the leads I and II, we will first express EF from the equation [2]:

\[ EF = II + ER \]  

and introduce it in the equation [10] as follows:

\[ a_{VF} = II + EL - \frac{1}{2}(EL + EL - I) \]  

From the equation [1] we can delineate the definition of ER as follows:

\[ ER = EL - I \]

The substitution of the ER in the equation [14] results in the following equations:

\[ a_{VF} = II + EL - I - \frac{1}{2}(EL + EL - I) \]  

\[ a_{VF} = II - \frac{1}{2} I \]

The leads aVR and aVF can be easily delineated in the same way:

\[ a_{VR} = -\frac{1}{2} I \]  
\[ a_{VL} = I - \frac{1}{2} II \]

For theoretical discussion we ignored the influence of the coordinate system on the value of EA, i.e., EA of 45 degree is actually -45 degrees (7).

With the use of the sinus theorem, we can easily calculate EA from the lead I and II (11):

\[ EA = \pm \arctan \left( \frac{2II - I}{\sqrt{3I}} \right) \]

One of the versions of this formula was used in the first study performed by Madanmohan et al (2). In the second publication from the same group (3) the following formula for the calculation of the EA was applied:

\[ EA = \pm \arctan \left( \frac{a_{VF}}{I} \right) \]

If we express the value of the lead II as a combination of values aVF and I from the equation [17]:

\[ II = a_{VF} + 1/2 I \]

and introduce it in the equation [20] the new formula for the calculation of EA would be:

\[ EA = \pm \arctan \left( \frac{2a_{VF}}{\sqrt{3I}} \right) \]

Similarly, expressing lead I as the combination of leads II and aVF (from equation [17]):

\[ I = 2I - 2a_{VF} \]

and introducing it in the formula [20] we get the formula for the calculation of EA from the leads II and aVF:

\[ EA = \pm \arctan \left( \frac{a_{VF}}{\sqrt{3(II - a_{VF})}} \right) \]

Results

Means and standard error of the mean (SEM) of leads I, II, and aVR are given in Table 1. Table 2 summarizes calculated EA. EA calculated from leads I and aVF with the use of our formula [23] gains similar results to those calculated from lead I and II (formula [20]) and from leads II and aVF (formula [25]): 37°±4°, 37°±3° and 37°±4°, respectively (p>0.05, for all combinations). However, the values of EA obtained with the formula [21], described by Madanmohan et al (3) were significantly lower (34°±4°) as compared to the formula [20], formula [23], and formula [25] (Table 2).
### Table 1: QRS wave amplitudes in 48 normal ECGs. [view this table]

### Table 2: Electrical axis (EA) in 48 normal hearts with EA between 0° and 90°. [view this table]

#### Discussion

The calculation of EA seems to be a trivial problem, but this premise is sometimes misleading. Using a previously published formula (2) we obtained comparable results. However, in the second study of the same author (3), a methodological error occurred: the calculation of EA from bipolar and unipolar leads was done without respecting the different strengths of bipolar and unipolar leads. Wilson et al (12) introduced unipolar leads and his approach was later modified by Goldberger (10). This modification resulted in increased amplitudes in unipolar leads, which are since then called augmented (e.g., augmented VR=aVR): the strengths of leads I, II, and III are different to those of aVR, aVL, and aVF.

Since different formulas give different results, we performed the analysis on the real data because the voltage depends on several factors (e.g., body impedance). We confirmed our theoretical considerations. The use of computers allows accurate and fast calculations and there should be no reasons to use wrong formulas, particularly if accuracy can be achieved with a simple correction. The basic value of EA is, therefore $\pm \arctan \left( \frac{2aVF}{\sqrt{3}I} \right)$ and NOT $\pm \arctan(aVF/I)$.

The absolute differences between results of different formulas are often of minor clinical relevance for a single patient. Nevertheless, a correct calculation of EA has great importance in epidemiological studies (13). Our recorder uses the maximal values of the leads amplitudes for the calculation of the EA, but is not necessarily the same QRS complex for which values of the waves are printed out. Due to this bias we were not able to compare our results with the EA calculated by the recorder itself.

More and more of ECGs are analyzed with computerized systems. Hence, we should be aware that those computer systems could be a source of erroneous calculations (14). Our results suggest that further studies should be performed to evaluate the accuracy of formulas implemented in ECG recorders.

#### Acknowledgment

The authors are grateful to Drs. R.P. Hof and P. Dirschedl for critical comments.

#### References


Recieved: September 14, 1998
Accepted: January 8, 1999

Correspondence to:
Dragutin Novosel
Externer Psychiatrischer Dienst
Haselstrasse 1
CH - 5401 Baden, Switzerland
dragonovosel@hotmail.com